

BIET, SIKAR
Vibration Engineering (GME4A)
I-Mid term Paper Solution

Solution

Q. 1 (a)

Solution 1- Auditory effect of noise -

- Annoyance - it creates annoyance to the receptors due to sound level fluctuations.
- Physiological effects - The physiological features like breathing amplitude, blood pressure, pulse rate are effected.
- Loss of hearing: Long exposure to high sound level cause loss of hearing.
- Human Performance: The working performance of workers will be affected as they'll be losing their concentration.
- Nervous System: It causes pain, ringing in the ears, feeling of tiredness thereby effecting the functioning of human system
- Sleeplessness: It affects the sleeping there by inducing the people to become restless and loose concentration.

Non-Auditory Effects of Noise On People -

- An increase in the number of people with psychological & physiological health problem requiring the increased use of certain types of drugs.
- An increase in the incidence of female infants with reduce gestation period and body weight at the time of birth.
- An increase in the number of adults requiring admission to psychiatric hospital.

Q1 b)

(i) Sound Power Level -

Sound Power is the rate at which acoustic energy is radiated from a sound source. It can be expressed in watt or decibel (dB)

Sound Power Level (PWL) is defined as the logarithmic ratio of the sound power emitted to a reference sound power

$$L_w = 10 \log_{10} \frac{\text{Sound Power}}{\text{Reference Sound Power}}$$

$$= 10 \log_{10} \frac{W}{W_0}$$

The internationally agreed reference power is 10^{-12} W.

i.e. $W_0 = 10^{-12}$ W

$$\therefore L_w = 10 \log_{10} W + 120 \text{ (dB)}$$

where \rightarrow

W = Sound power measured in watts

W_0 = Reference Sound Power = 10^{-12} W

(ii) Sound Intensity Level -

Sound intensity is a vector quantity

$$\text{Intensity} = \frac{\text{Power}}{\text{Area}}$$

In a free field environment i.e. no reflected sound wave and well away from any sound source, the sound intensity is related to the root mean square (rms) acoustic pressure as follows

$$I = \frac{P_{\text{rms}}^2}{\rho c}$$

ρ \rightarrow Density of air in kg/m^3

c \rightarrow speed of sound

ρc \rightarrow Acoustic impedance

(iii) Sound Pressure level -

The range of sound pressure that can be heard by the human ear is very large. The minimum acoustic pressure audible to the young human ear judged to be in good health and unaided by too much exposure to excessive loud music, is approximately 20×10^{-6} Pa.

The decibel is a logarithmic expression of power ratio. Sound power is proportional to, at free field,

$$L_p = 10 \log_{10} \left(\frac{P_{rms}^2}{p^2} \right) \text{ (dB)}$$

It should be noted that sound pressure related to the highest sound pressure level are still remarkably smaller than the static atmospheric pressure of about 10^5 Pa.

$$L_p = 20 \log_{10} \left(\frac{P_{rms}}{P} \right)_{ref} \text{ (dB)}$$

$$P_{rms} = \frac{P}{\sqrt{2}} = 0.707 P$$

$$P_{ref} = 2 \times 10^{-5} \text{ Pa}$$

$$\text{So, } L_p = 20 \log_{10} P_{rms} + 94 \text{ (dB)}$$

Q1 (a)

Industrial Noise Control -

The human ear respond differently to sound of different frequencies. The ear "hears" high frequency sound of a given level more loudly than low frequency sound of the same level. Research has shown that if sound is weighted so as to reduce the effect of low frequencies, then the resultant combined level of sound of all frequencies in the audible range 20 - 20,000 cycle per second, relate well to subjective experience. This weighting is called 'A-weighting' and the A-weighted sound level, dB(A) is commonly used for measurement of environmental sound in UK. It can be used to indicate the subjective human response to sound.

(b)

Solution -

$$x_1 = 4 \cos(\omega t + 10^\circ)$$

$$x_2 = 6 \sin(\omega t + 60^\circ)$$

Let $x = x_1 + x_2$

$$A \sin(\omega t + \alpha) = 4 \cos(\omega t + 10^\circ) + 6 \sin(\omega t + 60^\circ)$$

$$A \sin \omega t \cos \alpha + A \cos \omega t \sin \alpha = 4 \cos \omega t \cos 10^\circ - 4 \sin \omega t \sin 10^\circ + 6 \cos \omega t \sin 60^\circ + 6 \sin \omega t \cos 60^\circ$$

$$= \cos \omega t (4 \cos 10^\circ + 6 \sin 60^\circ) + \sin \omega t (-4 \sin 10^\circ + 6 \cos 60^\circ)$$

$$A \sin \omega t \cos \alpha + A \cos \omega t \sin \alpha = \cos \omega t (9.13) + \sin \omega t (2.3)$$

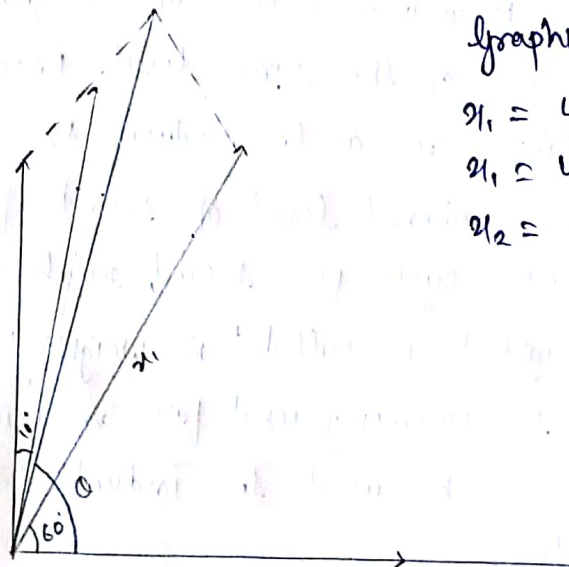
Comparing

$$A \cos \alpha = 2.3$$

$$A \sin \alpha = 9.13$$

$$A^2 = 88.74 \Rightarrow \boxed{A = 9.42}$$

$$\tan \alpha = 9.13/2.3 \Rightarrow \boxed{\alpha = 75.86}$$



Graphically Method

$$x_1 = 4 \cos(\omega t + 10^\circ)$$

$$x_2 = 6 \sin(\omega t + 60^\circ)$$

$$x = 9.42 \sin(\omega t + 75.86^\circ)$$

Q. (2) (a)

Solⁿ:- Compound Pendulum:- A system which is vertically suspended & oscillates with a small amplitude under the force of gravity is known as compound pendulum. It is also an example of single degree of freedom system. It is also called Physical pendulum.

Let

w = weight of rigid body

O = point of oscillation

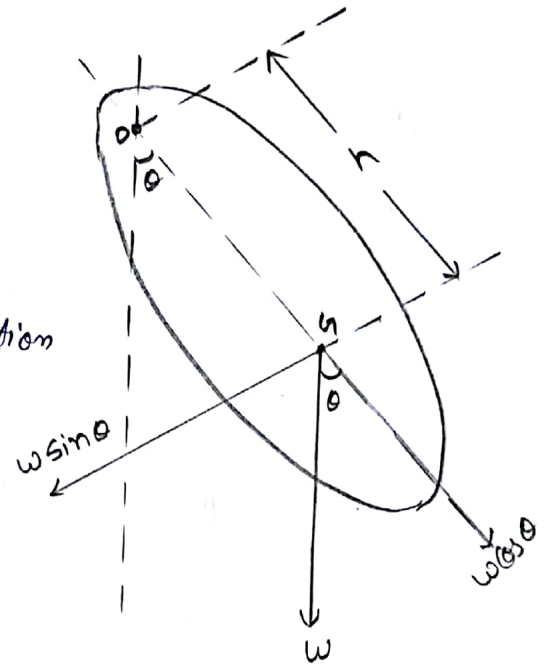
k = Radius of gyration about an axis through G

h = Distance from point of oscillation to the point G

G = Centre of Gravity

I = Moment of inertia about an axis parallel to the C.G.

$$I = mk^2 + mh^2$$



$$\begin{aligned} \text{Restoring torque} &= -w \sin \theta \times h = -mgh \sin \theta \\ &= -mgh \theta \end{aligned}$$

$$\text{Inertia torque} = I \ddot{\theta}$$

$$\therefore I \ddot{\theta} = -mgh \theta$$

$$I \ddot{\theta} + mgh \theta = 0$$

$$\ddot{\theta} + \frac{mgh}{I} \theta = 0$$

So

$$\omega_n = \sqrt{\frac{mgh}{mk^2 + mh^2}} = \sqrt{\frac{gh}{k^2 + h^2}}$$

$$f_n = \frac{1}{2\pi} \sqrt{\frac{gh}{k^2 + h^2}}$$

Q (2) (b)

Solⁿ:-

Potential energy of spring = $\frac{1}{2} kx^2$

Kinetic energy of mass 'm' = $\frac{1}{2} m\dot{x}^2$ (linear motion)

Kinetic energy of pulley = $\frac{1}{2} I \dot{\theta}^2$ (angular motion)

\therefore K.E. + P.E. = Const.

$$\frac{1}{2} I \dot{\theta}^2 + \frac{1}{2} m\dot{x}^2 + \frac{1}{2} kx^2 = C$$

$$\frac{1}{2} (\frac{1}{2} Mr^2) \dot{\theta}^2 + \frac{1}{2} m\dot{x}^2 + \frac{1}{2} kx^2 = C$$

differentiate with respect to time

$$\frac{1}{2} (\frac{1}{2} Mr^2) (2\dot{\theta}\ddot{\theta}) + \frac{1}{2} m r^2 (2\dot{\theta}\ddot{\theta}) + \frac{1}{2} k r^2 (2\dot{\theta}\ddot{\theta}) = 0$$

$$2 \times \frac{1}{2} r^2 \dot{\theta} [\frac{1}{2} M \ddot{\theta} + m \ddot{\theta} + k \theta] = 0$$

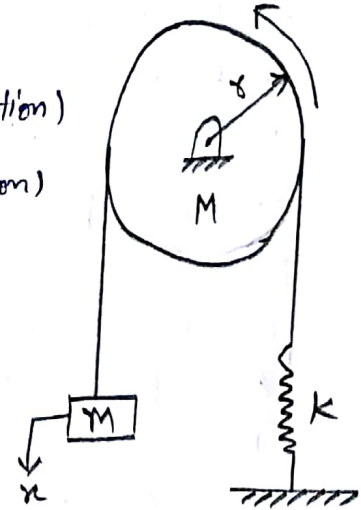
$$\frac{1}{2} M \ddot{\theta} + m \ddot{\theta} + k \theta = 0$$

$$(\frac{1}{2} M + m) \ddot{\theta} + k \theta = 0$$

$$\ddot{\theta} + \frac{k}{(\frac{1}{2} M + m)} \theta = 0$$

So

$$\omega_n = \sqrt{\frac{k}{\frac{1}{2} M + m}}$$



taking
 $x = r\theta$
 $I = \frac{1}{2} Mr^2$

Q.2 (a)

Solution 1- Viscous damping - when a system is allowed to vibrate in a viscous medium is k/as viscous damping.

Viscosity - viscosity is a property of fluid by virtue of which oppose the motion of a fluid.

According to Newton's law of motion

$$\tau \propto du/dy$$

$$\frac{F}{A} = \mu du/dy$$

$$F = \mu A \frac{du}{dy} \Rightarrow \boxed{F = C\dot{x}}$$

The eqⁿ of the motion for system is -

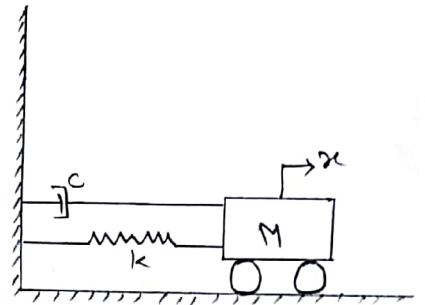
$$\boxed{M\ddot{x} + C\dot{x} + Kx = 0}$$

where,

$M\ddot{x}$ - acceleration force

$C\dot{x}$ - damping force

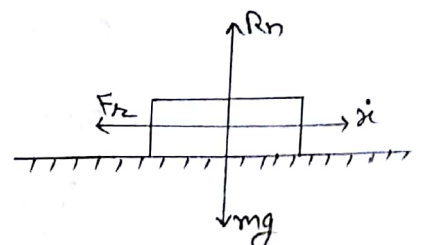
Kx - spring force



Coulomb damping - when one body is allowed to slide over the other, the surface of one body offer some resistance to the movement of the other body on it. This resistance force is called force of friction. Some amount of energy is wasted in overcoming this friction as the surfaces are dry. So, the amount of coulomb damping is -

$$F_d R_n \text{ or } \boxed{F_r = \mu R_n}$$

The friction force acts in a direction opposite of the direction of velocity.



Q.2. (b)

Solution 1- Given data -

$$m = 3 \text{ kg}, k = 100 \text{ N/m}, C = 3 \text{ N}\cdot\frac{\text{sec}}{\text{m}}$$

i) Damping ratio

$$\xi = C/C_c$$

$$\text{and } C_c = 2\sqrt{km} = 2\sqrt{100 \times 3} = 34.64 \text{ N}\cdot\frac{\text{sec}}{\text{m}}$$

$$\text{So, } \xi = \frac{3}{34.64} = 0.086 \quad \underline{\text{Ans}}$$

ii) Frequency of damped vibration

$$\omega_d = \omega_n \sqrt{1 - \xi^2}$$
$$= \sqrt{\frac{k}{m}} \times \sqrt{1 - \xi^2}$$

$$\omega_d = \sqrt{\frac{100}{3}} \times \sqrt{1 - (0.086)^2} = 5.74 \text{ rad/sec} \quad \underline{\text{Ans}}$$

$$f_d = \frac{1}{2\pi} \times \omega_d = \frac{1}{2\pi} \times 5.74 = 0.92 \text{ Hz} \quad \underline{\text{Ans}}$$

iii) Logarithmic decrement

$$\delta = \frac{2\pi\xi}{\sqrt{1 - \xi^2}}$$

$$\delta = \frac{2\pi(0.086)}{\sqrt{1 - (0.086)^2}} = 0.54 \quad \underline{\text{Ans}}$$

iv)

$$\delta = \frac{1}{n} \ln\left(\frac{x_1}{x_{n+1}}\right)$$

$$0.54 = \frac{1}{n} \ln\left(\frac{x_1}{\frac{20x_1}{100}}\right)$$

$$0.54 = \frac{1}{n} \ln\left(x_1 / \frac{x_1}{5}\right)$$

$$0.54 = \frac{1}{n} \ln(5)$$

$$n = 2.98 \approx 3 \quad \underline{\text{Ans}}$$